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NCC-003-027201

Seat No. _____

M. Sc. (ECI) (Sem. II) (CBCS) Examination

May / June – 2017

Paper-05 : ECI Mathematics - II

[Old Course]

Faculty Code : 003

Subject Code : 027201

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (i) **All** questions carry **equal** marks.

(ii) Figures on **right** hand side indicate marks.

1 Answer the following : (any seven) **14**

(1) Find gof and fog, if $f : R \to R$ and $g : R \to R$ are given by $f(x) = \sin x$ and $g(x) = x^3$ so that $gof \neq fog$.

(2) Convert $40^{o}\ 20'$ into Radian measure.

(3) Find the derivative at $x = 2$ of the function $f(x) = x^3$.

(4) Find the principle value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$.

(5) Find $\frac{d^2y}{dx^2}$, if $y = x^5 + \cot x$.

(6) Find the rate of change of the area of a circle per second with respect to its radius r when $r = 5$ cm.

(7) Find the integral of $\int (\tan x + \cot x)\, dx$.

(8) Find the value of i^{200}.

(9) Find the order and degree of $x\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^4-\frac{dy}{dx}=0$.

(10) Find the vector joining the points $P(2,3,0)$ and $Q(-1,-2,-4)$ directed from P to Q.

2 Answer the following : (any **two**) **14**

(1) Let $f:N\to R$ be a function defined as 7

$f(x)=4x^2+12x+15$. Show that $f:N\to S$, where S is the range of f, is invertible. Find the inverse of f.

(2) Solve the following in brief : 7

(i) $\sin x=\frac{\sqrt{3}}{2}$

(ii) $\sec x=2$

(3) Find the limit of $\lim\limits_{x\to 1}\left[\frac{x^2-4}{x-2}-\frac{1}{x^3-3x^2+2x}\right]$. 7

3 Answer the following : **14**

(1) Show that $\tan^{-1}x+\tan^{-1}\frac{2x}{1-x^2}=\tan^{-1}\frac{3x-x^3}{1-3x^2}, |x|<\frac{1}{\sqrt{3}}$. **5**

(2) Find the derivative of f given by $f(x)=\cos^{-1}x$ assuming it exists. **5**

(3) Find $\frac{dy}{dx}$: **4**

(i) If $x=a\tan\theta,\ y=a\cot\theta$

(ii) If $x=at^2,\ y=2at$.

OR

3 Answer the following : **14**

(1) Differentiate $x^{\left(x+\frac{1}{x}\right)}$ with respect to x. 7

(2) Find the approximate change in the volume V of a cube of side x meters caused by increasing the side by 2%. 7

4 Answer the following : **14**

(1) Find the integral 7

(i) $\int \frac{x+1}{\sqrt{2x+1}}\,dx$

(ii) $\int \sin^3 x \cos^3 x\,dx$

(2) Using integration find the area of region bounded by the triangle whose vertices are (1, 0), (2, 2) and (3, 1). 7

5 Answer the following : (any **two**) **14**

(1) In a bank, principal increases continuously at the rate of 5% per year. In how many years Rs. 1,000 double itself ? 7

(2) Find angle 'θ' between the vectors $\overline{a} = \hat{i} + \hat{j} - \hat{k}$ and $\overline{b} = \hat{i} - \hat{j} + \hat{k}$. 7

(3) Prove that $3\sin\frac{p}{6}\sec\frac{p}{3} - 4\sin\frac{5p}{6}\cot\frac{p}{4} = 1$. 7

(4) If $y = \left(x + \sqrt{x^2-1}\right)^m$, show that $\left(x^2-1\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = m^2 y$. 7